Analysis and Configuration of Boundary Difference Calculations

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Abstract. In the field of land management, stakeholders (people) everywhere have many disputes over the location of boundaries between private land and public land. We find that the stakeholders disagree with each other over boundaries. We propose an approach that helps people to come to an agreement on position of boundaries (including pixel-based approach, polygon-based approach and middle boundary approach). The experiments are carried out on data relating to public parks in Auckland, New Zealand. The results of the experiments highlight the differences between different stakeholder's percieved boundaries.

Keywords: Boundary Negotiation, Boundary Disputes, Polygon-based Boundary Calcluation, GPS-based Boundary Detection

1 Introduction

Land Management is the process of managing the use and development (in both urban and rural settings) of land resources. Land resources include organic agriculture, reforestation, water resource management and eco-tourism projects, is called public land. Private property is a legal designation of the ownership of property by non-governmental legal entities. The use of public land for private purposes, known as encroachment, has been identified as a problem affecting public parks in the Auckland region. Every boundary conflict contains a strong spatial component [1]. The spatial location of a public park is defined by its boundary. However there may be several different versions of the same boundary for a park. Perception of encroachment depends upon the viewpoint of the stakeholder. The stakeholders are residents living around the parks, non-residents who use the parks, organised sports groups and representatives of the Auckland City Council (including managers, councillors and surveyors). In land management, arguments occur over boundaries between stakeholders.

2 Related Work

This sections examines research work that has been carried out in related areas. The related areas are the mathematical formulas used to calculate the differences between two sets of data (boundary or area) and the magnitude of the differences. A search of the literature reveals that people use the Hidden Markov Model [2] [3],the Boundary Element Method [4] [5] and point-set-based [6] [7] [8] to detect differences in boundaries. Hidden Markov models (HMM) are studied for the purpose of planar shape classification using curvature coefficients. A discrete-time HMM is a probabilistic model that describes a random sequence as the indirect observation of an underlying (hidden) random sequence where this hidden process in Markovian. The boundary element method (BEM) is a numerical computational method of solving linear partial differential equations which have been formulated as integral equations. A point-set-based model is developed for areal objects from a perspective that incorporates spatial cognition. This model is called with point-set-based regions (PSBR). Computing spatial relationships between two PSBRs using the derived areal objects consists of looking at topological relationships, directional relationships, metric relationships, distance between centroids, average distance and Hausdorff distance.

2.1 Motivation

After studying the methods listed above, we have found that they give an accurate detection of the differences. However, these works does not really solve the arguments among stakeholders, due to these methods not providing a possible solution for the stakeholders. We analysed the pixel-based calculation and proposed a polygon-based calculation to form a new point of view for the stakeholders, upon which they can base negotiations to solve the boundary dispute.

3 Methodology

Land use conflict occurs whenever land-use stakeholders have incompatible interests related to land areas that result in negative effects [9]. In order to resolve the arguments over boundaries, we examine two existing approaches: a) Pixel-based Approach and b) Polygon-based Approach and we then build upon the two approaches above to propose a new approach: Middle-boundary Approach.

3.1 Pixel-based approach

Given a set of n sequential GPS coordinate pairs $G = \{(Lo_1, La_1), \ldots, (Lo_n, La_n)\}$ for one area, we firstly transfer them into integer coordinates according to certain predefined precision, for example $C = \{(X_1, Y_1), \ldots, (X_n, Y_n) \mid X_i = round(Lo_i \times 1000), Y_i = round(La_i \times 1000)\ \forall i \in \{1, \ldots, n\}\}$. Then we shift the coordinate origin somehow to fit the coordination set as,

$$C_s = \{(x_1, y_1), \ldots, (x_n, y_n) \mid x_i = X_i - min(X) + 1, y_i = Y_i - min(Y) + 1 \\ \forall i \in \{1, \ldots, n\}\} \tag{1}$$

Now we have a bitmap with n positive pixels. Next we sequentially connect each neighboring pair (n-th point is the neighbor with $n-1$-th and 1-st). For example, if have (x_i, y_i) and (x_{i+1}, y_{i+1}), we need to compute a set of pixels approximately connecting (x_i, y_i) to (x_{i+1}, y_{i+1}) and also as a edge of closed polygon. To achieve such closed approximate pixel edge, we simply approximate the y coordination from a continuous series of x connecting x_i to x_{i+1} and do the reversed, as follows (assume $x_i \le x_{i+1}$ and $y_i \le y_{i+1}$)

$$\begin{aligned} &x = [x_i, x_i + 1, \ldots, x_{i+1}] \\ &y = y_i + x \times \tfrac{y_{i+1} - y_i}{x_{i+1} - x_i} \\ &y = round(y); \end{aligned} \tag{2}$$

$$\begin{aligned} &y = [y_i, y_i + 1, \ldots, y_{i+1}] \\ &x = x_i + y \times \tfrac{x_{i+1} - x_i}{y_{i+1} - y_i} \\ &x = round(x). \end{aligned} \tag{3}$$

The result with more points from (2) and (3) is taken as the edge point set between (x_i, y_i) and (x_{i+1}, y_{i+1}). Once all edges for one area if obtained, we can apply area fill algorithm to fill the edge graph and obtain a binary bitmap of that area. For area difference, we can simply do a matrix subtraction to compute following areas:

1. both A_1 and A_2 covers;
2. A_1 covers but A_2 not;
3. A_2 covers but A_1 not.

As any field area is represented by a binary matrix $M = \{0, 1\}^{m \times n}$, where m and n denote the number of rows and columns respectively of the bitmap, thus the area is simply calculated by

$$S = sum(sum(M)). \tag{4}$$

3.2 Polygon-based Approach

In the analytic geometry method, a boundary is seen as a polygon formed by connecting points sequentially. Given two boundaries $A = \{a_1, a_2 \ldots a_n\}$ and $B = \{b_1, b_2 \ldots b_m\}$, where the end points of A and B are clockwise distributed. There are four steps to find the difference $A - B$ (i.e. the area inside A but outside B) and $B - A$ (i.e. the area inside B but outside A).

1. Find all cross points C between any edge pairs, one from A and one from B. As each edge is a line segment, there are many existing algorithms for finding cross point between two given line segments. The result of this step is a set of points $C = c_1, c_2, \ldots c_o$, each cross point c_k is associated with one edge $a_i a_{i+1}$ from A and one edge $b_j b_{j+1}$ from B.
2. Form the difference polygons $D_1, D_2, \ldots, D_o$. As boundary A have o cross points with B, it is easy to imagine that there are o difference polygons. And the k-th difference polygon D_k is defined by cross point c_k, a sequence of points A_k from A, cross point c_{k+1} and a sequence of points B_k from B.

Note that when $k = o$ have $c_{k+1} = c_1$, as the 'next' for the last one in a circle is the first. Then, to determine the difference polygon D_k, we need to find out the sequences A_k and B_k. To compute the sequences A_k, we need firstly check if c_k and c_{k+1} are associated with the same edge $a_i a_{i+1}$ in A. If so, means the boundary B cross edge $a_i a_{i+1}$ at least twice and there is no end point from A between c_k and c_{k+1}, thus we have $A_k = \emptyset$. If not, say c_k associates with $a_i a_{i+1}$ and c_{k+1} associates with $a_l a_{l+1}$, then A_k is the sequences $[a_{i+1}, a_{i+2}, \ldots, a_l]$. Also note that if have $i + 1 > l$, the sequences A_k actually becomes $[a_{i+1}, a_{i+2}, \ldots, a_n, a_1, \ldots, a_l]$, always keep in mind that we are working on a 'circle'. Applying the same method, we can determine the sequences B_k say $B_k = [b_{j+1}, b_{j+2}, \ldots, b_p]$. Recall that both A and B are clockwise distributed, so are the A_k and B_k. Thus to define the difference polygon D_k, we need to use the inverse sequences $B'_k = [b_p, \ldots, b_{j+2}, b_{j+1}]$ with is counterclockwised. By now we have the difference polygon $D_k = c_k A_k c_{k+1} B'_k$ computed. And so for all difference polygons $D_1, D_2, \ldots, D_o$

3. Determine which set of difference polygons belong to $A - B$ and which set to $B - A$. It is easy to conclude that for any neighboring difference polygon D_k and D_{k+1} should have the different identity, as if boundary of A is 'outside' of B between cross points c_k and c_{k+1} which indicates D_k belongs to $A-B$, after the cross in c_{k+1}, the boundary of A becomes 'inside' the boundary which indicates that D_{k+1} belongs to $B - A$, and vice versa. Thus, we can simply determine the difference polygons belonging by finding out the identity of any difference polygon. This can be done by detect whether a point from A (e.g. a_1) is inside polygon B. Make a ray from a_1 to any direction, if there is odd numbered cross point with polygon B, then a_1 is inside B; and if the cross number is even, then a_1 is outside B.
4. Compute the area of difference polygons. Given any polygon $A = a_1 a_2 \ldots a_n$ and its coordinate set $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$, the area of A can be computed as

$$S_A = \frac{1}{2} \sum_{i=1}^{n} (x_i y_{i+1} - x_{i+1} y_i). \tag{5}$$

Note that when $i = n$, have $a_{n+1} = a_1$ and $x_{n+1} = x_1, y_{n+1} = y_1$.

3.3 Middle-boundary Approach

To find the middle-boundary, we propose a nearest neighbor based algorithm. Let $A = a_1 a_2 \ldots a_n$ be the boundary with more endpoints than $B = b_1 b_2 \ldots b_m$, we calculated a middle boundary $M = m_1 m_2 \ldots m_n$ have the same number of end points with A, each m_i lays in the halfway of a_i to its nearest neighbor in $\{b_1, b_2, \ldots, b_m\}$. To find such nearest neighbor, we need firstly compute a distance vector $D = R^{m \times 1}$ have

$$D_j = dist(a_i, b_j) = \sqrt{(x_{ai} - x_{bj})^2 + (y_{ai} - y_{bj})^2}. \tag{6}$$

Then find the j with smallest D_j, and eventually compute the mean vector of $[x_{ai}, y_{ai}]$ and $[x_{bj}, y_{bj}]$.

Algorithm 1 Find the Middle Boundary

Require: Polygon $A = a_1 a_2 \ldots a_n$ and $B = b_1 b_2 \ldots b_m$, have $n >= m$; the coordinate of each point $\{(x_{ai}, y_{ai}) | \forall i \in [1, n]\}$, $\{(x_{bj}, y_{bj}) | \forall j \in [1, m]\}$

Ensure: The middle boundary $M = m_1 m_2 \ldots m_n$ and its coordinate set $\{(x_{mi}, y_{mi}) | \forall i \in [1, n]\}$.

for $i \in [1, n]$ **do**

  Current point is a_i, (x_{ai}, y_{ai});

  Find the current point's nearest neighbor b_j (x_{bj}, y_{bj}) from B;

  Compute the m_i, have $x_{mi} = mean(x_{ai}, x_{bj})$, and $y_{mi} = mean(y_{ai}, y_{bj})$

end for

Once M is determined, using algorithm mentioned in Section3.2, we can compute the difference area between A to M and B to M.

4 Experiments

To demonstrate the boundary comparison techniques, for this paper, 20 parks from the Manukau and North Shore areas are selected as representative examples of the four categories of park discussed above.

4.1 Experimental setup

A number of experiments were conducted to study the effectiveness of the new approach. In this section we look at the data used in the experiments, the setup and procedure of the experiments and we discuss the results of the experiments.

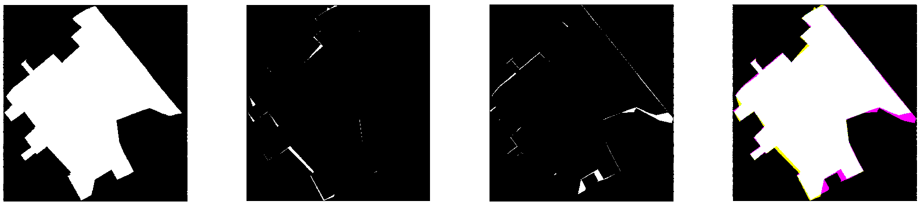
4.2 Location and Device

GPS Data Collection carried out at 20 sites across North Shore and Manukau areas in Auckland. One GPS devices used: Leica Viva TPS. The data collected, using the Leica Viva TPS device, uses the Mt Eden 2000 co-ordinates system.

4.3 Data

The data used in the experiments is boundary data of New Zealand specifically the Auckland area.

1 GPS Boundary Data. The GPS boundary data has been collected during field visits to the selected parks.
2 Land Boundary Data. The Council boundary data is used to compare against the observed boundary data.



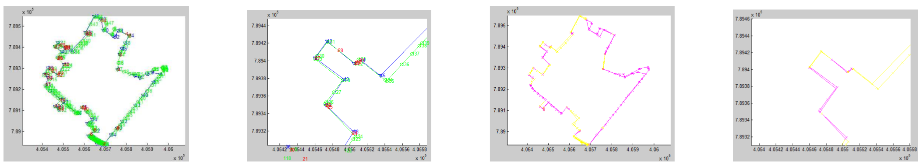
(a) Area covered by both sets of data

(b) Positive differences between data sets

(c) Negative differences between data sets

(d) Composite image of data

Fig. 1: Pixel Based Approach



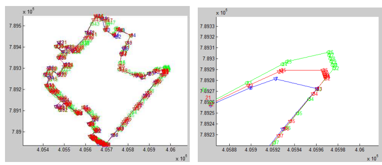
(a) Boundary cross points

(b) Boundary cross points - zoomed in view

(c) Differences between polygons

(d) Differences between polygons - zoomed in view

Fig. 2: Polygon Based Approach



(a) Overview of Middle-Boundary Approach

(b) Zoomed-in view of Middle-Boundary Approach

Fig. 3: Middle-Boundary Approach

4.4 Discussion

The argument from people for the analyzing frames illuminates the underlying causes of Park boundries in the geographic analysis of public land use. Geographic information systems (GIS) have long been applied in resolving municipal/local boundary conflicts (e.g. US political redistricting) [10]. This approach utilises both GIS and GPS in presenting and resolving boundary disputes.

Table 1: Results of the polygon-based experiments

	Polygon-based				Middle-boundary	
Name	AreaOmG	AreaGmO	AreaOmM	AreaMmO	AreaGmM	AreaMmG
Agincourt Reserve	3050.970703	46.8359375	2566.961914	20.89550781	132.734375	590.8027344
Aorere Park	4635.09082	2241.937531	2224.094147	1173.392487	1138.012573	2480.46402
Auburn Reserve	12983.03125	0.544921875	7507.915039	111.6933594	0.13671875	5586.398438
Beaumont Park	351.6609802	854.4945984	291.2216797	363.8186035	596.5552368	166.3186951
Dale Reserve	0	0	0	0	13001.17609	3857.525452
Diana Reserve	234.1083984	179.5517578	118.3222656	93.94726563	88.72363281	118.9091797
Gallaher Park	1846.82312	2653.692627	1193.333954	1283.261169	1471.670105	754.7279663
Holland Reserve	2972.285156	319.7568359	1480.804688	9044.454102	502.2460938	10718.42871
James Watson Park	240.3355103	15420.63184	120.597168	6829.786682	9515.397919	1044.291077
Jellicoe Park	549.1130066	481.79776	483.4128418	275.4145508	272.3617249	131.6789246
Killarney Park	3160.15918	8609.833008	1402.926758	1428.557617	7316.314453	1892.266602
McFetridge Park	5461.837891	103.5576172	4819.770508	36.79882813	1404.405273	1979.706055
Normanton Reserve	544.5351563	1909.788086	354.5947266	373.9892578	1688.322266	342.4638672
Puhinui Domain	19511.99994	69455.7518	56116.37057	82893.59921	2093.553741	28110.97137
Robert White Park	0	0	0	0	0	0
Russell Road Reserve	1511.267151	9524.483978	1052.96701	7910.964752	1634.870209	479.6509399
Stancich Reserve	824200.4697	1560491.801	64279.37207	2873.81543	819018.793	50225.59277
Tadmoor Park	8388.599365	3037.15976	3787.05957	733.4524231	11932.83618	1146.519043
Taharoto Park	743.9765625	37836.19238	321.2314453	37842.42578	323.984375	38055.57227
Teviot Reserve	297.4169922	69.4296875	406.0068359	78.33398438	263.484375	163.7978516

The results of the pixel-based experiments show the differences between two views (sets of boundary data) for the same area. The first set of differences shows the encroachment of private land onto public land and the second set of differences shows the encroachment of public land onto private land. This approach highlights the differences between the boundaries but does not propose any possible solutions to the problem. The results of the polygon-based experiments in Table 1 show the plotting of the two sets of points and highlights the differences between the two views (sets of boundary data) for the same area. AreaOmG gives the area of the original boundary minus that of the GPS boundary. AreaGmO gives the area of the GPS boundary minus that of the original boundary. AreaOmM gives the area of the original boundary minus that of the middle boundary. AreaMmO gives the area of the middle boundary minus that of the original boundary. AreaGmM gives the area of the GPS boundary minus that of the Middle boundary. AreaMmG gives the area of the middle boundary minus that of the GPS boundary. The results of the experiments show the differences between two views (sets of boundary data) for the same area. We give a fair solution to people who have arguments on the measurement of boundary for Parks in Auckland, as seen in Fig. 3. The green line shows the boundary as perceived by the council and the blue line shows the boundary as perceived by the results of a field survey. The differences in the two boundaries show where the boundary is disputed. The red line shows a calculated middle boundary which may act as a starting point for resolving boundary disputes. Stakeholders have different views on encroachment. For example the council have a strict viewpoint and assume that the data they have is correct whereas some private residents have a relaxed viewpoint on the position of a boundary.

5 Conclusions and Future Work

In order to address the land encroachment problems in Auckland's parks, we firstly proposed two different approaches boundary calculations. Though both of them detect and highlight differences, in numerical terms, effectively, whereas neither approach offers a possbile solution to any boundary conflicts. We then proposed the middle-boundary approach, in which we address the boundary arguments from stakeholders by a nearest neighbor based algorithm. This solution could be a possible solution that disputes or at least a starting point for negotiations, because the (middle-boundary) have been addressed. The main limitation of the middle-boundary approach is that the proposed new boundary is based solely on mathematical calculations and does not take into account the stakeholders' views or motivations currently. Possible future work could involve adding weighting to the middle-boundary calculations so that different possible solutions could be generated.

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